Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Wednesday 19 June 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3A**

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. (a) Given that

$$y = \ln\left(3 + x^2\right)$$

complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 significant figures.

x	2	2.5	3	3.5	4	4.5	5
y	1.946	2.225		2.725	2.944	3.146	3.332

(1)

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(b) Use Simpson's rule with all the values of y in the completed table to estimate, to 3 significant figures, the value of

$$\int_2^5 \ln\left(3 + x^2\right) dx$$

(3)

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_2^5 \ln\sqrt{\left(3 + x^2\right)}\, dx$$

(1)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. Use algebra to determine the values of x for which

$$\left|x^2 - 2x\right| \leqslant x$$

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

3. Use L'Hospital's rule to show that

$$\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right) = 0$$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4.

The Taylor series expansion of $\mathrm{f}(x)$ *about* $x = a$ *is given by*

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

The curve with equation $y = \mathrm{f}(x)$ satisfies the differential equation

$$\cos x \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y^2 \frac{\mathrm{d}y}{\mathrm{d}x} + \sin x = 0$$

Given that $\left(\frac{\pi}{4}, 1\right)$ is a stationary point of the curve,

(a) determine the nature of this stationary point, giving a reason for your answer.

(2)

(b) Show that $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \sqrt{2} - 2$ at this stationary point.

(4)

(c) Hence determine a series solution for y, in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^3$, giving each coefficient in simplest form.

(2)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5.

$$y = e^{3x} \sin x$$

(a) Use Leibnitz's theorem to show that

$$\frac{d^4y}{dx^4} = 28e^{3x} \sin x + 96e^{3x} \cos x$$

(6)

(b) Hence express $\frac{d^4y}{dx^4}$ in the form

$$Re^{3x} \sin(x + \alpha)$$

where R and α are constants to be determined, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

(3)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where a and b are positive constants.

Given that

- the eccentricity of H is the reciprocal of the eccentricity of E
- the coordinates of the foci of H are the same as the coordinates of the foci of E

determine

(i) the value of a

(ii) the value of b

(6)

Question 6 continued

(Total for Question 6 is 6 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $t = \tan\left(\frac{\theta}{2}\right)$ to show that

$$\int \frac{1}{2\sin\theta + \cos\theta + 2}\,\mathrm{d}\theta = \int \frac{a}{(t+b)^2 + c}\,\mathrm{d}t$$

where a, b and c are constants to be determined.

(3)

(b) Hence show that

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{1}{2\sin\theta + \cos\theta + 2}\,\mathrm{d}\theta = \ln\left(\frac{2\sqrt{3}}{3}\right)$$

(4)

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Question 7 continued

(Total for Question 7 is 7 marks)

P 7 5 6 8 6 R A 0 1 9 3 2

Turn over

8. The parabola P has equation $y^2 = 4ax$, where a is a positive constant.

The point $A\left(at^2, 2at\right)$, where $t \neq 0$, lies on P.

(a) Use calculus to show that an equation of the tangent to P at A is

$$yt = x + at^2$$

(3)

The point $B(2k^2, 4k)$ and the point $C(2k^2, -4k)$, where k is a constant, lie on P.

The tangent to P at B and the tangent to P at C intersect at the point D.

Given that the area of the triangle BCD is 432

(b) determine the coordinates of B and the coordinates of C.

(5)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 8 marks)

9. (i) The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 13 \\ 5 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point P.

(a) Determine the coordinates of P.

(2)

Given that the plane Π contains both l_1 and l_2

(b) determine a Cartesian equation for Π.

(4)

(ii) Determine a Cartesian equation for each of the two lines that

- pass through (0, 0, 0)
- make an angle of 60° with the x-axis
- make an angle of 45° with the y-axis

(4)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10. The motion of a particle P along the x-axis is modelled by the differential equation

$$t^2\frac{d^2x}{dt^2} - 2t(t+1)\frac{dx}{dt} + 2(t+1)x = 8t^3e^t \qquad \text{(I)}$$

where P has displacement x metres from the origin O at time t minutes, $t > 0$

(a) Show that the transformation $x = tu$ transforms the differential equation (I) into the differential equation

$$\frac{d^2u}{dt^2} - 2\frac{du}{dt} = 8e^t$$

(4)

Given that P is at O when $t = \ln 3$ and when $t = \ln 5$

(b) determine the particular solution of the differential equation (I)

(8)

Question 10 continued

Question 10 continued

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Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS